

Algebra, Number Theory and Combinatorics

Overall

Problem 1. Let G be a finite group. Let k be a field of positive characteristic p . Show that the following assertions are equivalent:

- (1) G is a p -group;
- (2) for every non-zero k -representation V of G , $V^G \neq 0$.

Problem 2. (a) Construct a diffeomorphism $GL_3(\mathbb{R}) \rightarrow O(3) \times \mathbb{R}^6$.

(b) Propose an analogous statement for $GL_n(\mathbb{C})$, and prove it.